Continuous optimization
PGE305

Laurent Pfeiffer
Inria and CentraleSupélec, Université Paris-Saclay

Ensta-Paris
Institut Polytechnique de Paris
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Exercise 1

The function f is defined by

$$f(x,y) = \frac{x^3}{3} + \frac{x^2}{2} + 2xy + \frac{y^2}{2} - y + 9.$$

We observe that

$$f(x,0) = \frac{x^3}{3} + \frac{x^2}{2} \underset{x \to -\infty}{\longrightarrow} -\infty.$$

Therefore, the problem has no global solution.

We have

$$\nabla f(x,y) = \begin{pmatrix} x^2 + x + 2y \\ 2x + y - 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \iff \begin{cases} x^2 + x + 2(1-2x) = 0 \\ y = 1 - 2x \end{cases}$$

$$\iff \begin{cases} x^2 - 3x + 2 = 0 \\ y = 1 - 2x \end{cases} \iff \begin{cases} (x,y) = (1,-1) \\ \text{or } (x,y) = (2,-3). \end{cases}$$

Exercise 1

There are two stationary points: $(1, -1)$ and $(2, -3)$. We have

$$D^2 f(\bar{x}) = \begin{pmatrix} 2x+1 & 2 \\ 2 & 1 \end{pmatrix}.$$

We recall that for a matrix of the form $M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$,

- M is positive definite $\iff a > 0$ and $ac - b^2 > 0$
- M is positive semi-definite $\iff a \geq 0$ and $ac - b^2 \geq 0$.

- The point $(1, -1)$ is not a local minimizer since,

$$D^2 f(1, -1) = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

  is not positive semi-definite (the determinant is negative).

- The point $(2, -3)$ is a local minimizer since

$$D^2 f(2, -3) = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$$

  is positive definite (upper diagonal term and determinant positive).