Continuous optimization
PGE305

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Exercise 2

Lagrangian:

$$L(x, y, \lambda) = \frac{1}{2}(x - x_0)^2 + \frac{1}{2}(y - y_0)^2 - \lambda_1(1 - x - y) - \lambda_2 x - \lambda_3 y.$$

KKT conditions:

- Stationarity:

$$\nabla L(x, y, \lambda) = \begin{pmatrix} x - x_0 + \lambda_1 - \lambda_2 \\ y - y_0 + \lambda_1 - \lambda_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Sign condition: $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, $\lambda_3 \geq 0$.
- Complementarity condition: $x + y < 1 \implies \lambda_1 = 0$, $x > 0 \implies \lambda_2 = 0$, and $y > 0 \implies \lambda_3 = 0$.

Exercise 2

Case 1: $(x_0, y_0) = (1, 1)$.
Solution: $(x, y) = (1/2, 1/2)$.

- Complementarity: $x > 0 \implies \lambda_2 = 0$, $y > 0 \implies \lambda_3 = 0$.
- Stationarity:

$$\nabla L(x, y, \lambda) = \begin{pmatrix} -1/2 + \lambda_1 \\ -1/2 + \lambda_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

  for $\lambda_1 = 1/2$.
- Sign: $\lambda_1 \geq 0$.

Exercise 2

Case 2: $(x_0, y_0) = (0, 2)$.
Solution: $(x, y) = (0, 1)$.

- Complementarity: $y > 0 \implies \lambda_3 = 0$.
- Stationarity:

$$\nabla L(x, y, \lambda) = \begin{pmatrix} \lambda_1 - \lambda_2 \\ -1 + \lambda_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

  for $\lambda_1 = \lambda_2 = 1$.
- Sign: $\lambda_1 \geq 0$, $\lambda_2 \geq 0$.

Exercise 3

Case 2: $(x_0, y_0) = (-1, -1)$.
Solution: $(x, y) = (0, 0)$.

- Complementarity: $x + y < 1 \Longrightarrow \lambda_1 = 0$.
- Stationarity:

$$\nabla L(x, y, \lambda) = \begin{pmatrix} 1 - \lambda_2 \\ 1 - \lambda_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

  for $\lambda_2 = \lambda_3 = 1$.
- Sign: $\lambda_2 \geq 0$, $\lambda_3 \geq 0$.