Continuous optimization
PGE305

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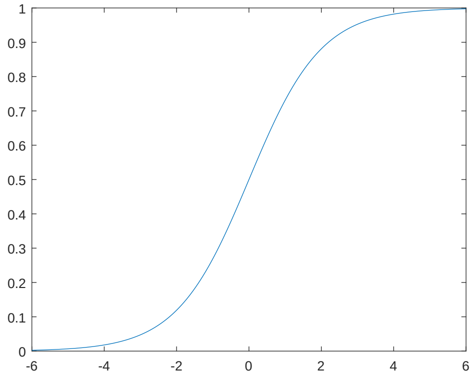
Exercise 6

1. Graph of P (for $t_0 = 0$, $\alpha = 1$, $p = 1$).



Exercise 6

Interpretation of the coefficients.

- p is the limit of the population: $p = \lim_{t \to +\infty} P(t)$.
- t_0 is a transition time: $P(t_0) = p/2$.
- α is a transition rate: $P'(t_0)/p = \alpha/4$.

2. Let us denote by (t_i, P_i) the pairs time-population provided as a data of the problem. The least-square problems writes:

$$\inf_{(p,t_0,\alpha)\in\mathbb{R}^3} \sum_{i=1}^{N} \left(y_i - \frac{p}{1 + exp(-\alpha(t_i - t_0))} \right)^2.$$

3. One can reasonably take: $p = 500$ and $t_0 = 2000$. We have: $P'(t_0) \approx (300 - 0)/(2000 - 1900) = 3$. Thus

$$\alpha = \frac{4P'(t_0)}{p} \approx \frac{12}{500} = 0,024.$$

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Solution: $p = 440,834,\ \alpha = 0,0216059,\ t_0 = 1976,63.$