Continuous optimization
PGE305

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Exercise 9

Let $x \in \mathbb{R}$. We consider

$$\min_{z \in \mathbb{R}} f(z), \quad \text{subject to: } \begin{cases} z \geq x, \\ z \geq -x. \end{cases}$$

Case 1: $x \geq 0$. Let $\bar{z} = x$. We have

$$\bar{z} \geq x \quad \text{and} \quad \bar{z} \geq 0 \geq -x.$$

Therefore $\bar{z}$ is feasible. Moreover, for any feasible $z \in \mathbb{R}$,

$$f(z) = z \geq x = \bar{z} = f(\bar{z}).$$

Therefore $\bar{z}$ is optimal.

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Case 2: $x \leq 0$. Let $\bar{z} = -x$. We have

$$\bar{z} \geq 0 \geq x \quad \text{and} \quad \bar{z} \geq -x.$$

Therefore $\bar{z}$ is feasible. Moreover, for any feasible $z \in \mathbb{R}$,

$$f(z) = z \geq -x = \bar{z} = f(\bar{z}).$$

Therefore $\bar{z}$ is optimal.

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Original problem:

$$\inf_{(x_i)_{i=1,\dots,N}} \sum_{i=1} |x_i - y_i| + \alpha \sum_{i=2}^{N} (x_i - x_{i-1})^2.$$

Reformulated problem:

$$\inf_{\substack{(x_i)_{i=1,\dots,N} \\ (z_i)_{i=1,\dots N}}} \sum_{i=1}^{N} z_i + \alpha \sum_{i=2}^{N} (x_i - x_{i-1})^2,$$

$$\text{subject to: } \begin{cases} z_i \geq x_i - y_i, & \forall i = 1, \dots, N \\ z_i \geq y_i - x_i & \forall i = 1, \dots, N. \end{cases}$$

Exercise 9

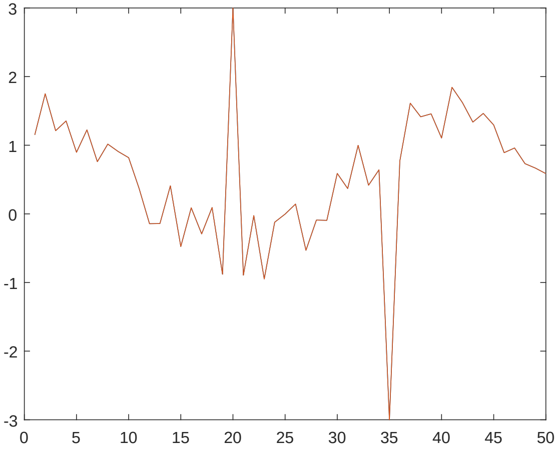

Figure: Noisy signal

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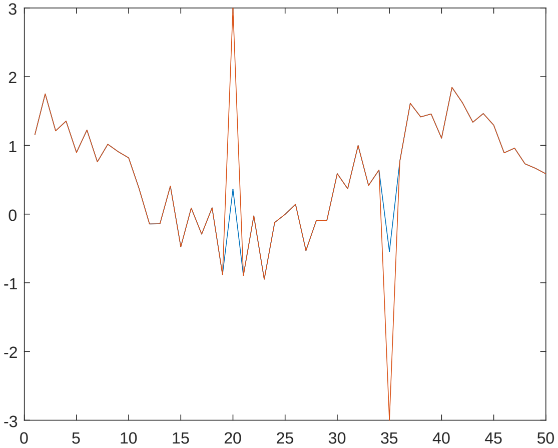

Figure: Signal denoising with $\alpha = 0.2$

Exercise 9

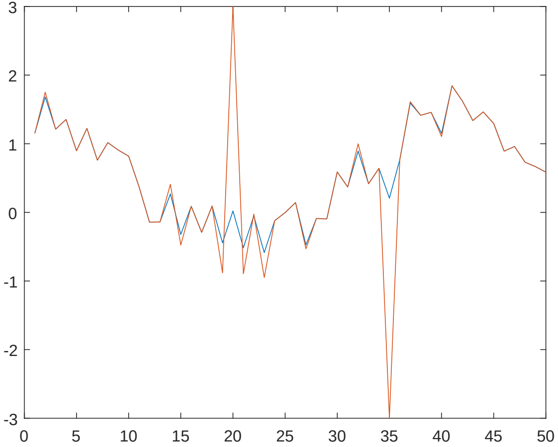

Figure: Signal denoising with $\alpha = 0.5$

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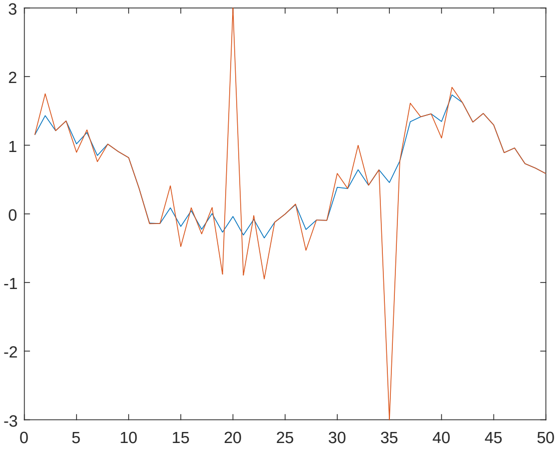

Figure: Signal denoising with $\alpha = 1$

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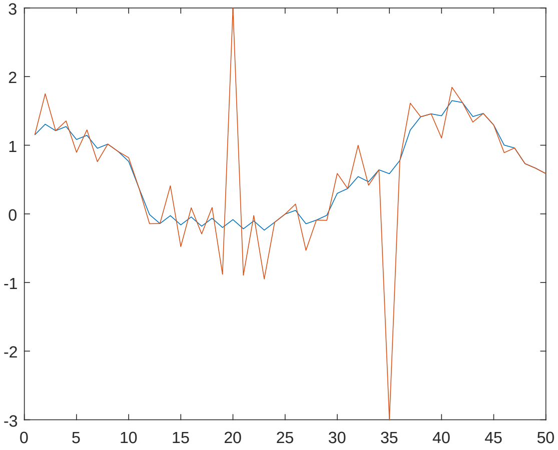

Figure: Signal denoising with $\alpha = 2$

Exercise 9

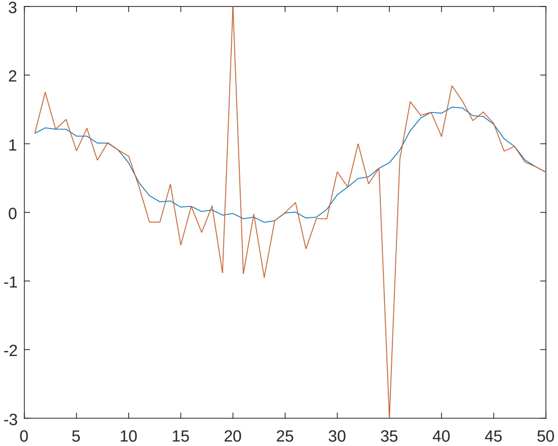

Figure: Signal denoising with $\alpha = 5$